ISE Cryptography – Lecture 02

Stream Ciphers

The story so far...

- Classical cryptography!
  - Don't use it in the modern world, though.
- Cryptanalysis!
  - Cracking classical ciphers (and appreciating awful alliteration).
- Perfect security!
  - Awesome, but about as useful as a chocolate teapot.
- Semantic security!
  - Less-than-perfect, but practical.
- The next logical pieces of the puzzle? Today's themes:
- How can we actually use a short key to securely encrypt data?
- How can we demonstrate that it's secure?
- Can we encrypt variable-length messages?
- All of this (and more) as we look at stream ciphers...

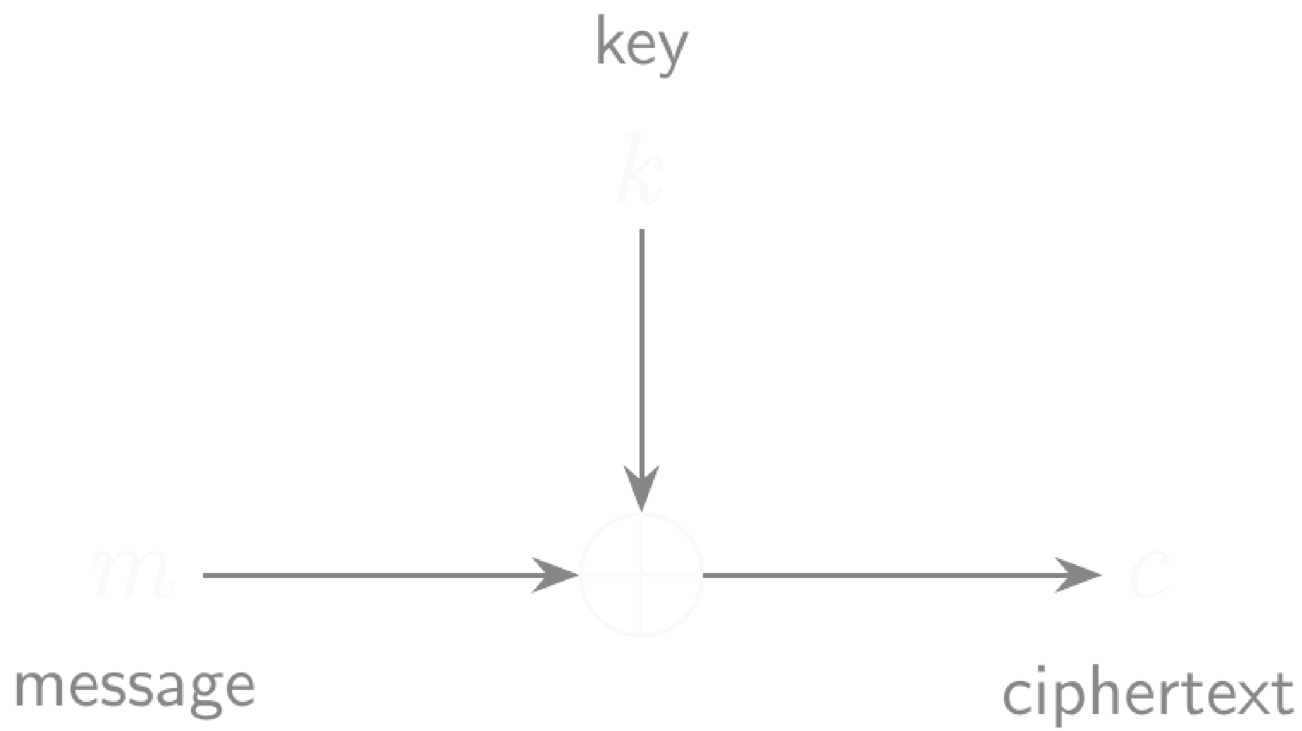
Fix the One-Time Pad?

Starting the day with some déjà vu

The One-Time Pad

- Everyone remember the one-time pad from last time?
- What does it use as...
  - The encryption function?
  - The decryption function?
  - The key?
- What constraints apply when using it?
- Is it...
  - Perfectly secure?
  - Semantically secure?
  - IND-CPA secure?
  - Secure against message recovery attacks?
- Why can't we just use it for everything and skip the rest of cryptography?

One-Time Pad: Encryption



$$c = E(k, m) = k \oplus m$$

The Key Problem

- Needing a key as long as the message makes the one-time pad almost useless.
  - How can we share it securely?
  - And it's only good for a single message!
  - The USSR (Venona) and Microsoft (PPTP) both fell victim to that particular issue!
- We need a way to use a short, fixed-length key to encrypt/decrypt
  - But we can't do this with the one-time pad!
  - Why can't we just repeat the short key end-to-end for long messages?
- If an attacker can predict parts of the key...
  - They can break semantic security!
- How did you generate keys for last week's lab/tutorial?
  - Probably with Python's `secrets` module, right?
  - So we can generate them, but we'd still have to share them...

The Integrity Problem

- A passive attacker can't read messages encrypted with a one-time pad...
  - But an active attacker can cause a lot of trouble.
  - The one-time pad is malleable
- You have no way of verifying whether the message you've decrypted is the one that was sent! It could have been...
  - Modified in transit
  - Stored and replayed later
  - Cut into chunks and selectively reassembled
- Also vulnerable to bit-flipping attacks
  - Flipping a bit in the ciphertext flips that same bit in the plaintext
- An attacker can make predictable changes to the plaintext...
  - Even if they can't read it!
- Extremely dangerous if the format of the message is known

Solutions

- We're going to leave the integrity/malleability problem alone for now.
  - We'll circle back and address it fully later on!
- Let's focus on the key problem.
  - Ideally, we want to work with short, manageable keys.
  - But we also want to be able to encrypt very long messages.
- We've got two broad paths we could go down:
  - Chop long messages into smaller chunks
  - Stretch short keys into longer keys
- Both of these are viable options!
  - Block ciphers encrypt fixed-length blocks of data
  - Stream ciphers produce a keystream from a short key
- These definitions look pretty clear right now...
  - ...but they'll blur later: we can also use block ciphers to produce a keystream!

Stream Ciphers

Building ciphers for the real world.

And now for something completely different...

- You might have heard of a little-known indie game about mining and crafting.
- Minecraft worlds are absolutely gigantic
  - 60,000,000 by 60,000,000 blocks
  - Procedurally generated terrain and other features
- How much data is needed to generate an entire world?
  - Just the seed
  - Either a 64-bit integer or text converted to a 32-bit integer
- Even a small change in the seed can create radically different terrain!
- Random, but also not random
  - It looks and feels random, but it’s completely deterministic
  - An identical seed will always generate the same world
  - It’s pseudorandom

Pseudo-Random Generators

- A **pseudo-random generator** (PRG) is an efficient, deterministic algorithm G
  - Also abbreviated as PRNG - **pseudo-random number generator**
  - Input: a seed s from a finite **seed space** $\mathcal{S}$
  - Output: a value r from a finite **output space** $\mathcal{R}$
  - $G : \mathcal{S} \to \mathcal{R}$
- Typically, $\mathcal{S}$ and $\mathcal{R}$ are sets of fixed-length bitstrings
  - Seed lengths are typically much shorter than output lengths
- Note that a PRG has to be efficient!
  - Where has that term come up before?
- And a PRG has to be deterministic.
  - Each input maps to a fixed output - it's not random!
- Intuitively, even though the function is deterministic...
  - ...the output should seem to be random!

True RNGs

- A random number generator that's actually random?
  - This is actually harder than you'd think!
- Classical computers aren't good at randomness
- Instead, we pull from a physical source of randomness
- Any suggestions for random physical processes?
- Onboard hardware, e.g. device drivers, disk activity, network traffic
  - Often the seed source used for `/dev/random`
- Quantum mechanics, e.g. radioactive decay, shot noise or qbits
- Thermal processes, e.g. Nyquist noise
- Oscillator drift, timing events, lava lamps...

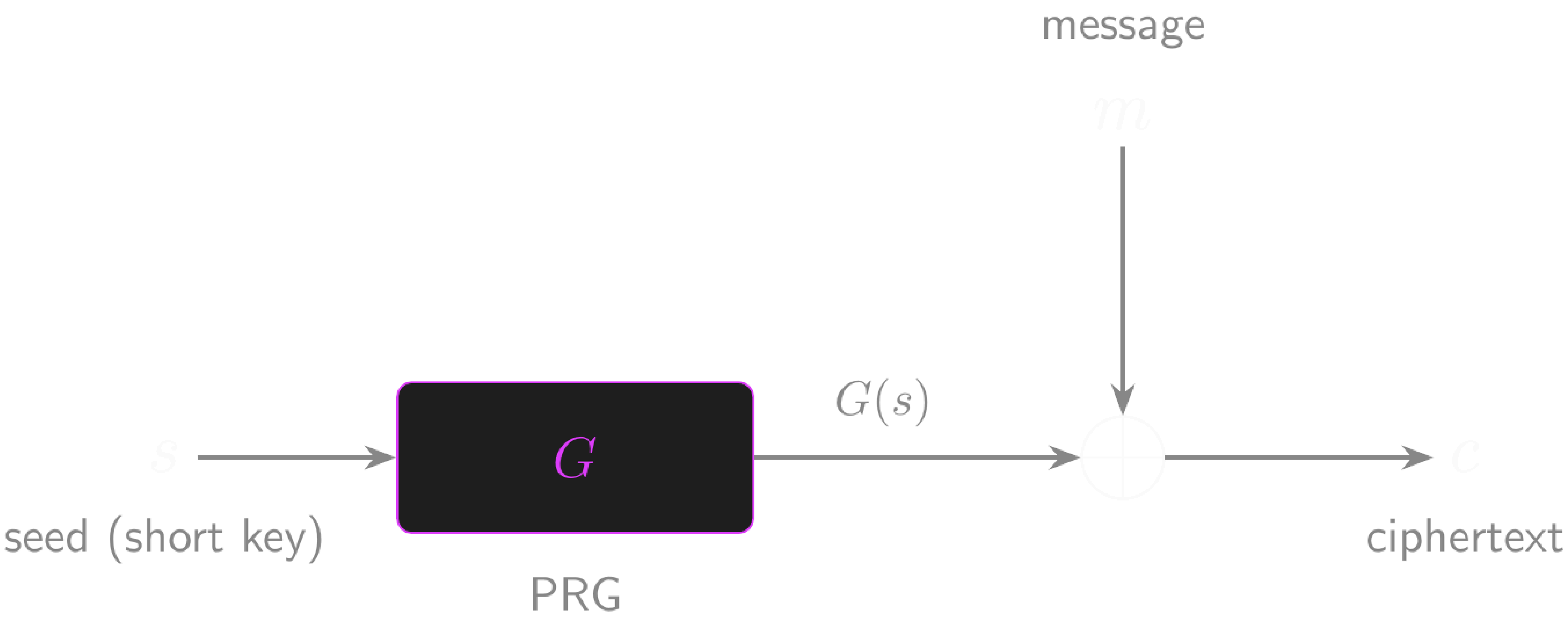
The Obligatory Lavarand Photo

- Cloudflare uses a wall of 100 lava lamps as an entropy source
  - A camera captures images of the lamps continuously
  - The chaotic, unpredictable flow of wax generates random data
  - Fed into their CSPRNG to seed cryptographic keys
- Why lava lamps? They’re a physical process that’s genuinely unpredictable
  - Tiny variations in heat, air currents, and wax composition
  - No two frames are ever the same
- Other companies use different approaches
  - Random.org uses atmospheric noise
  - Some use radioactive decay or quantum processes

Building a Stream Cipher

- How does this help us turn the one-time pad into a more practical cipher?
- Let's encrypt messages up to some maximum length L
  - How long does the key need to be if we use the original one-time pad?
- Let's pick a short seed value s with length $\ell < L$
- Assume that we have a PRG G that maps from $\mathcal{S}$ to $\mathcal{R}$
  - $\mathcal{S} = \{0,1\}^\ell$ and $\mathcal{R} = \{0,1\}^L$
- What if we replace the key with a pseudo-random number from G?
- Our stream cipher is defined as:
  - $E(s,m) = G(s) \oplus m$
  - $D(s,c) = G(s) \oplus c$
- Does the correctness property hold? Why (or why not)?
- Is it possible for this cipher to be perfectly secure?
- Is it possible for this cipher to be semantically secure?

Stream Cipher: Encryption



$$c = E(s, m) = G(s) \oplus m$$

Secure PRGs

- Think about this from the attacker's point of view...
  - The security of the cipher hinges on the properties of the PRG
  - Can we distinguish between the PRG's output and true random values?
  - Maybe! It depends™ on the PRG we're using.
- If we can't tell the difference between $G(s)$ and a truly random k...
  - With better accuracy than random guessing...
  - Then the one-time pad and our stream cipher can't be distinguished...
  - So our stream cipher must be secure!
- What's the catch?
  - This is semantic security, not perfect security!
  - And it only holds if the PRG is actually secure
  - How do we formalise what "secure PRG" means?

PRG Security Intuition

- Let's get some clear ideas about what makes a **secure PRG** for cryptography
- Start by picking a seed s at random from the seed space $\mathcal{S}$
  - Then pick an output r at random from the output space $\mathcal{R}$
  - And compute $G(s)$, the PRG's output for the chosen seed
- $G(s)$ and r should be computationally indistinguishable
  - No efficient adversary should be able to effectively tell the difference
  - If that holds, the PRG is secure!
- This might sound pretty familiar...
  - Indistinguishable?
  - Efficient adversary?
- Let's build a quick guessing game...

PRG Security: Attack Game

- This time around, our attack game will have two sub-games or **experiments**
  - The adversary $\mathcal{A}$ doesn't know which one they're playing, and has to guess!
- Experiment 0: the challenger computes...
  - $s \xleftarrow{R} \mathcal{S}$
  - $r \leftarrow G(s)$
- Experiment 1: the challenger computes...
  - $r \xleftarrow{R} \mathcal{R}$
- r is sent to $\mathcal{A}$, and $\mathcal{A}$ outputs its guess of experiment 0 or 1
- Let W_b be the probability that $\mathcal{A}$ outputs 1 on Experiment b
  - $\mathrm{PRG_{adv}}[\mathcal{A}, G] = |\Pr[W_0] - \Pr[W_1]|$
- If $\mathrm{PRG_{adv}}[\mathcal{A}, G]$ is negligible for all $\mathcal{A}$, then G is a **secure PRG**

PRG Security: Attack Game

Experiment 0

$s \xleftarrow{R} \mathcal{S}$

$r \leftarrow G(s)$

Experiment 1

$r \xleftarrow{R} \mathcal{R}$

Adversary $\mathcal{A}$

Receives r

Outputs $\hat{b} \in \{0,1\}$

$$\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G] = |\Pr[W_0] - \Pr[W_1]|$$

where $W_b = \Pr[\mathcal{A} \text{ outputs } 1 \text{ in Exp } b]$

PRG Security: Experiment 0

Experiment 0: r is **PRG output**

Challenger

Adversary $\mathcal{A}$

$s \xleftarrow{R} \mathcal{S}$

$r \leftarrow G(s)$

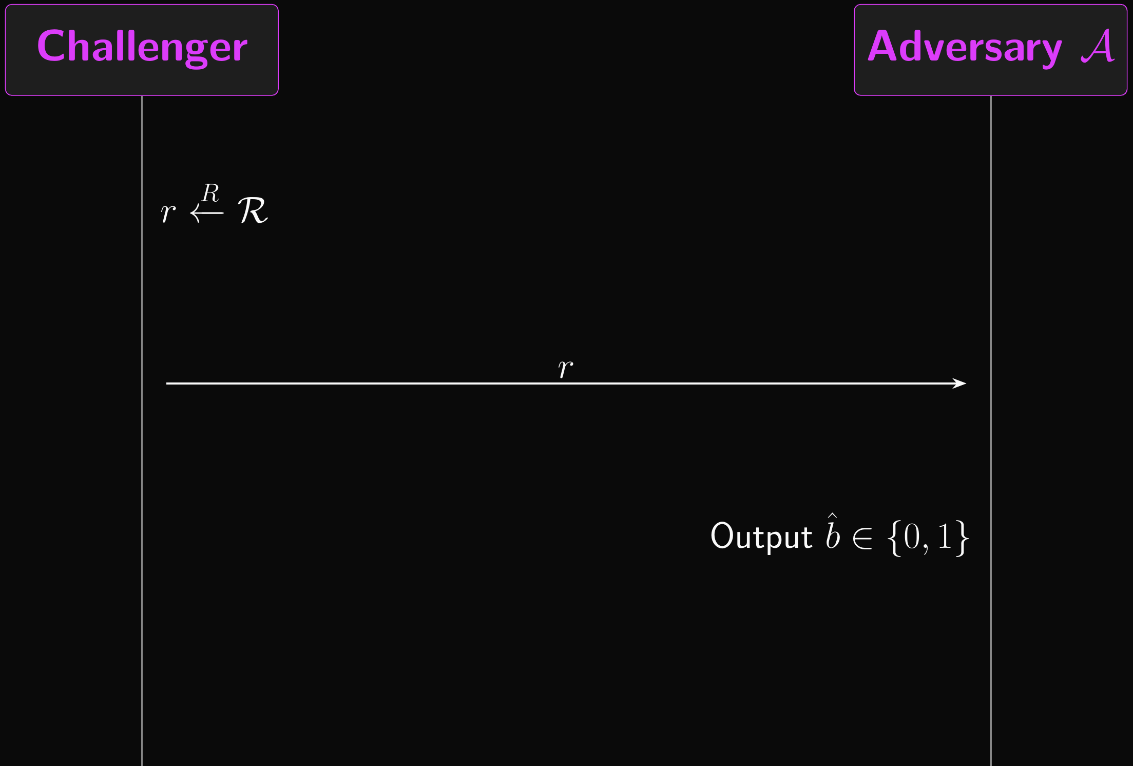
$\xrightarrow{\quad r \quad}$

Output $\hat{b} \in \{0, 1\}$

$W_0 = \Pr[\mathcal{A} \text{ outputs } 1]$

PRG Security: Experiment 1

Experiment 1: r is **truly random**



$$W_1 = \Pr[\mathcal{A} \text{ outputs } 1]$$

How Big is Big Enough?

- There's one more element needed to make our PRG secure
  - Why was it so easy to break the Caesar cipher?
- An adversary can always try to brute-force the seed space
  - Try every possible seed $s \in \mathcal{S}$, compute $G(s)$, check if it matches
  - How long does this take? $|\mathcal{S}|$ operations (one per seed)
- If $\ell = 4$: $|\mathcal{S}| = 2^4 = 16$ seeds. Trivial to brute-force.
- If $\ell = 40$: $|\mathcal{S}| = 2^{40} \approx 10^{12}$. A fast computer checks this in minutes.
- If $\ell = 128$: $|\mathcal{S}| = 2^{128} \approx 10^{38}$. At a billion guesses per second, this takes 10^{29} seconds. The universe is only $\approx 4 \times 10^{17}$ seconds old.
- The seed length ℓ controls how hard it is to break the PRG
  - This is a really important idea, so we need to formalise it!

The Security Parameter

- The **security parameter** λ is the single number that controls how secure a system is
  - In symmetric crypto, λ is the key or seed length in bits
  - In our stream cipher, the seed length ℓ plays the role of λ
  - Everything is parameterised by λ: key spaces, seed spaces, advantage bounds
- We write $\{0,1\}^\lambda$ for the seed/key space, giving $|\mathcal{S}| = 2^\lambda$ possible seeds
- Increasing λ makes life harder for efficient adversaries
  - The work to brute-force grows as 2^λ (exponential)
  - But legitimate operations (encryption, decryption) stay polynomial in λ
  - This gap is what makes computational security possible!
  - But a computationally unbounded adversary can always brute-force, no matter how large λ is
- $\lambda = 128$ is the standard minimum for symmetric cryptography today
  - $\lambda = 256$ gives a margin against future advances (including Grover's algorithm)
- When we say an advantage is "negligible", we mean negligible **as a function of λ**

Negligible and Super-Poly

- A function $f(\lambda)$ is **negligible** if it shrinks faster than any inverse polynomial in λ
  - For every positive integer d, there exists λ_0 such that for all $\lambda > \lambda_0$: $f(\lambda) < 1/\lambda^d$
  - Intuition: not just small, but *vanishingly* small. Smaller than $1/\lambda$, $1/\lambda^2$, $1/\lambda^{100}$, ...
  - Example: $1/2^\lambda$ is negligible. It beats any $1/\lambda^d$ once λ is large enough.
- The mirror image: $Q(\lambda)$ is **super-poly** if it grows faster than any polynomial in λ
  - Q is super-poly iff $1/Q$ is negligible
  - Otherwise, Q is **poly-bounded**
  - Example: 2^λ is super-poly. No polynomial λ^d can keep up.
- Now we can state security precisely:
  - "For all efficient adversaries $\mathcal{A}$, $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G]$ is negligible in λ"

Seed Space

- The seed space is $\mathcal{S} = \{0,1\}^\lambda$, so $|\mathcal{S}| = 2^\lambda$
- 2^λ is super-poly in λ, so the brute-force probability $1/2^\lambda$ is negligible
- A PRG can only be secure if the seed space is super-poly
  - Otherwise, an efficient adversary can just enumerate it!
- Given a large enough security parameter λ, our PRG is secure!
- All the security definitions we've seen work the same way:
  - $\mathrm{SS}_{\mathrm{adv}}$, $\mathrm{PRG}_{\mathrm{adv}}$, and later $\mathrm{BC}_{\mathrm{adv}}$, ... all must be negligible in λ

Building a Secure Stream Cipher

- Our stream cipher $\mathcal{E}$ is defined as:
  - $E(s, m) = G(s) \oplus m$
  - $D(s, c) = G(s) \oplus c$
- What's the only difference between the two?
  - Stream cipher uses a PRG to generate the key
  - One-time pad uses a truly random key
- Secure PRG output is indistinguishable from a random bitstring
  - An adversary can only do negligibly better than random guessing
  - No useful information about the seed is leaked from the keystream
- To have any non-negligible chance at distinguishing ciphertexts...
  - ...an adversary has to exploit some pattern in the PRG output!
- The security of a stream cipher is based on $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G]$
  - If $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G]$ is negligible, then $\mathrm{SS}_{\mathrm{adv}}[\mathcal{B}, \mathcal{E}]$ is negligible
- But why is this true? Let's prove it!

Theorems and Lemmas

- Before we dive in, let's define some terms you'll see in the textbook
- A **theorem** is a statement that has been proven to be true
  - "If the PRG is secure, the stream cipher is semantically secure" is a theorem!
  - It has a specific structure: "if [assumptions], then conclusion"
- A **lemma** is a smaller result that helps prove a bigger theorem
  - Think of it as a stepping stone: useful on its own, but mainly there to make the main proof easier
  - We'll prove lemmas and then use them as building blocks
- A **proof** shows *why* a theorem is true, not just *that* it is
  - In cryptography, most proofs are constructive: we build something (an adversary, a simulator) to demonstrate the claim
- You won't need to memorise proofs for the exam
  - But understanding the *technique* behind each proof is important: these patterns recur throughout cryptography

Proof by Reduction

- Our first proof technique: the **reduction**
- The claim: if the PRG G is secure, then the stream cipher $\mathcal{E}$ built from G is semantically secure
- How do we prove this? By showing:
  - "If you could break the stream cipher, you could use that ability to break the PRG"
  - But we assumed the PRG is secure (nobody can break it!)
  - So nobody can break the stream cipher either
- This is proof by contradiction, with a twist:
  - We don't just say "assume it's broken"
  - We build a specific adversary $\mathcal{B}$ that *uses* the stream cipher attacker $\mathcal{A}$ as a subroutine
  - $\mathcal{B}$ translates $\mathcal{A}$'s ability to break $\mathcal{E}$ into an ability to break G

The Reduction

- Adversary $\mathcal{B}$ plays the PRG game: receives a challenge string r
  - r is either $G(s)$ for a random seed s, or a truly random string
  - $\mathcal{B}$ doesn't know which!
- $\mathcal{B}$ uses r to run an SS game against $\mathcal{A}$:
  - $\mathcal{A}$ submits two messages m_0, m_1
  - $\mathcal{B}$ flips a coin $b \xleftarrow{R} \{0, 1\}$ and computes $c \leftarrow r \oplus m_b$
  - $\mathcal{B}$ sends c to $\mathcal{A}$, who outputs a guess $\hat{b}$
  - $\mathcal{B}$ outputs 1 ("it was a PRG") if $\hat{b} = b$, else 0 ("it was random")
- **If** $r = G(s)$: $\mathcal{A}$ sees a real stream cipher ciphertext and wins with its usual advantage
- **If** r **is truly random**: $\mathcal{A}$ sees a one-time pad ciphertext, and its advantage is 0!
- Therefore: $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G] \geq \mathrm{SS}_{\mathrm{adv}}[\mathcal{A}, \mathcal{E}]$
  - If the left side is negligible (PRG is secure), the right side must be too

What Just Happened?

- We proved stream cipher security *without knowing anything about how G works*
  - We only used the fact that G is a secure PRG
  - This is the power of reductions: they let us build on existing guarantees
- The key trick: replacing $G(s)$ with a truly random string turns the stream cipher into a one-time pad
  - Real world (PRG output): adversary has some advantage ϵ
  - Ideal world (truly random): adversary has advantage 0 (perfect security!)
  - The gap between the two worlds is exactly the PRG advantage
- This “real world vs ideal world” pattern shows up in nearly every security proof in this course

Ghosts of Problems Past

- We're flying it, so let's fix the length problem while we're here!
  - We can encrypt arbitrary-length messages shorter than the keystream.
  - $E(s, m) = G(s)[0 \ldots |m| - 1] \oplus m$
  - $D(s, c) = G(s)[0 \ldots |c| - 1] \oplus c$
- Have we solved the key length problem?
  - Yes!
- Have we solved the malleability/integrity problem?
  - No!
  - But we'll get back to that one... eventually.
- Key reuse was a big problem for the one-time pad
  - Does our stream cipher allow for key reuse?
  - Try it out and see!
  - We'll see some possible solutions to this later

PRGs all the way down...

Making generators from generators.

Building PRGs from PRGs

- Our PRG G maps $\{0,1\}^\lambda \to \{0,1\}^L$, producing L bits of keystream
  - But what if our message is longer than L bits?
  - Do we need to design a completely new PRG with a bigger output?
- We can build larger PRGs from existing ones!
  - Two approaches: parallel and sequential composition
  - Both produce $n \times L$ bits of output from a PRG that only outputs L bits
  - But they have very different tradeoffs
  - And we have to be careful not to compromise the security of the PRG!

Parallel Construction

- One technique is n**-wise parallel composition**
  - We take n independent seeds…
  - Generate the output from each seed…
  - And concatenate the results into one long output
- The new PRG, G', is defined over $(\mathcal{S}^n, \mathcal{R}^n)$
  - $G'(s_1, \ldots, s_n) = (G(s_1), \ldots, G(s_n))$ for $(s_1, \ldots, s_n) \in \mathcal{S}^n$
  - The **repetition parameter**, n, must be poly-bounded
- Is it secure? Can an adversary distinguish it from truly random output?
  - Intuitively, an adversary has better odds the more we use the PRG
  - $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G'] \leq n \cdot \mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G]$
  - Security degrades linearly as the repetition parameter increases

Parallel Construction: Seed Size

- Note the seed is now n times as long: $(s_1, \ldots, s_n) \in \mathcal{S}^n$
  - We need $n \times \lambda$ bits of seed to get $n \times L$ bits of output
  - The expansion rate barely improves! Can we do better? (Yes, but first...)
- Easy to parallelise: each $G(s_i)$ is independent, so all n can be computed simultaneously
- But why does the security bound hold? Let's prove it!

Proving the Parallel Bound

- We claimed $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G'] \leq n \cdot \mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G]$. But why?
- We can't directly use a reduction to a single PRG instance (there are n of them!)
- Instead, we use a **hybrid argument**: build a chain of intermediate distributions
  - Each adjacent pair in the chain differs in exactly one slot
  - If an adversary can tell the endpoints apart, they must be able to tell some adjacent pair apart
  - And telling an adjacent pair apart is the same as breaking a single PRG instance!

The Hybrid Chain

- Define $n + 1$ hybrid distributions:
  - H_0: $(G(s_1), G(s_2), \ldots, G(s_n))$ – all real PRG outputs
  - H_1: $(r_1, G(s_2), \ldots, G(s_n))$ – first slot replaced with random
  - H_2: $(r_1, r_2, G(s_3), \ldots, G(s_n))$ – first two slots random
  - $\vdots$
  - H_n: $(r_1, r_2, \ldots, r_n)$ – all truly random
- The adversary needs to distinguish H_0 (all PRG) from H_n (all random)
- But H_0 and H_n are connected by a chain of n small steps
  - Each step changes exactly one slot from PRG output to truly random

Each Step is a PRG Game

- Suppose an adversary can distinguish H_i from H_{i+1}
  - These differ in only one slot: slot $i+1$ is $G(s_{i+1})$ in H_i and random r_{i+1} in H_{i+1}
- We can use this to break G itself:
  - Receive challenge r from the PRG game (either $G(s)$ or truly random)
  - Fill in slots $1, \ldots, i$ with fresh random values
  - Put the challenge r in slot $i+1$
  - Fill in slots $i+2, \ldots, n$ with fresh PRG outputs
  - If $r = G(s)$: this looks like H_i. If r is random: this looks like H_{i+1}.

Completing the Proof

- But we assumed G is a secure PRG!
  - No efficient adversary can distinguish $G(s)$ from random
  - So no efficient adversary can distinguish H_i from H_{i+1} for any i
  - And if they can't spot any single step, they can't spot the overall change from H_0 to H_n
- The total advantage is at most the sum of the n individual advantages
  - Each individual step has advantage $\leq \mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G]$
  - So the total is $\leq n \cdot \mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G]$
- This is why n must be poly-bounded! If n were super-poly, $n \cdot \epsilon$ could stop being negligible

The Hybrid Argument

- The hybrid argument is one of the most common proof techniques in cryptography
- Recipe for showing two distributions are indistinguishable:
  1. Build a chain of hybrids from one distribution to the other
  2. Make each adjacent pair differ in exactly one “atomic” step
  3. Show that each atomic step reduces to a known hard problem
  4. The total advantage is at most the sum of all the individual step advantages
- Think of it like a game of “spot the difference”:
  - Comparing two photos that differ in 10 places at once is hard
  - But if you had 11 photos, each differing from the next in only one place, you could check each pair
  - If nobody can spot any single change, nobody can spot the overall difference
- We’ll use this technique again when we look at block cipher modes and IND-CPA security

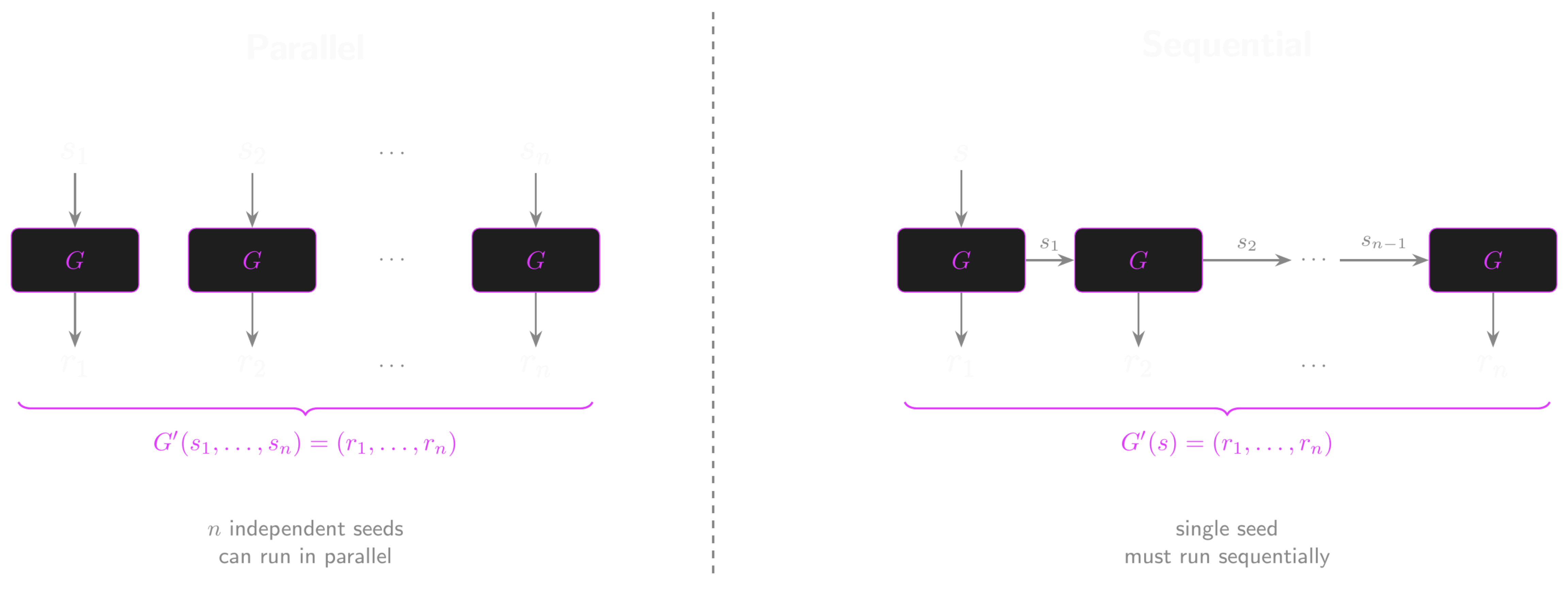
Sequential Construction

- The Blum-Micali method allows us to chain PRGs sequentially.
- This time, take G to be a secure PRG defined over $(\mathcal{S}, \mathcal{R} \times \mathcal{S})$
  - Outputs a seed in addition to the usual output!
- The new PRG, G', is defined over $(\mathcal{S}, \mathcal{R}^n)$
- $G'(s)$:
  - $s_0 \leftarrow s$
  - $\textbf{for } i = 1 \textbf{ to } n \textbf{ do: } (r_i, s_i) \leftarrow G(s_{i-1})$
  - $\textbf{return } (r_1, \ldots, r_n)$

Sequential Construction: Properties

- G' is the n**-wise sequential composition** of G
- The big win: only ONE seed of λ bits, but $n \times L$ bits of output!
  - Much better expansion rate than parallel composition
  - This is essentially how real stream ciphers work
- Downside: inherently sequential (each step needs the previous seed)
  - But ChaCha20 cleverly avoids this by using a counter instead of chaining seeds
  - The counter gives both sequential structure (for security) and random access (for parallelism)
- The security bound is similar to the parallel case (also provable by hybrid argument)
  - $\mathrm{PRG}_{\mathrm{adv}}[\mathcal{A}, G'] \leq n \cdot \mathrm{PRG}_{\mathrm{adv}}[\mathcal{B}, G]$

PRG Composition: Parallel vs Sequential



Expansion Rate

- We can evaluate PRGs by their expansion rate
- How much does it “stretch” the seed into the output?
- A PRG with a λ-bit seed and L-bit output has an expansion rate of L/λ
- Or we can use the seed space and output space to express it as
  - $\log|\mathcal{R}|/\log|\mathcal{S}|$
- Parallel composition: $n \times L$ bits of output from $n \times \lambda$ bits of seed
  - Expansion rate: $(n \times L)/(n \times \lambda) = L/\lambda$ – no improvement!
- Sequential composition: $n \times L$ bits of output from just λ bits of seed
  - Expansion rate: $(n \times L)/\lambda$ – scales linearly with n!

Real-World Stream Ciphers

From state-of-the-art to cautionary tale.

Let’s Build!

- Usually, this would be the part where we implement some kind of toy stream cipher
  - Great for a hands-on example, but riddled with flaws
- But I’ve been impressed so far… so let’s build a state-of-the-art stream cipher!
- And when I say stream cipher…
  - I mean a pseudo-random generator
  - With an XOR operation tacked on at the end
- Obligatory reminder: DON’T ROLL YOUR OWN CRYPTO
  - We’re implementing a cipher to learn by doing!
  - The cipher is secure
  - Your implementation isn’t necessarily secure!

Cryptographic Nonce

- Remember: the basic stream cipher $E(s, m) = G(s) \oplus m$ is deterministic
  - Encrypting the same message twice produces the same ciphertext!
  - It's only safe for a single message per key
- A **nonce** ("number used only once") breaks that determinism
  - Generally doesn't have to be secret (unlike a key)
  - Generally doesn't have to be unpredictable (unlike an IV)
  - But it does have to be unique!
- We'll encounter IVs (initialization vectors) next week!
  - Some sources will also call IVs nonces, so caveat lector.

Nonces and Multi-Message Security

- Adding a nonce turns a PRG into a pseudorandom function
  - A much more powerful cryptographic primitive!
  - Generates multiple outputs for a single seed by using different nonces
- This allows multiple messages to be safely encrypted with the same key!
  - Only if it’s a secure pseudorandom function, of course
  - And only if each key-nonce pair is used only once
  - Nonce reuse breaks security!
- Each key-nonce pair produces a unique keystream
  - This is the difference between single-message and multi-message security
  - We’ll formalise this as IND-CPA security next week with block cipher modes

Salsa and ChaCha

- We're going to take a different approach to the parallel and sequential compositions we looked at earlier.
- Instead, we're going to build a pseudorandom function in counter mode.
- Salsa and ChaCha are related families of fast stream ciphers.
  - Used as part of protocols like TLS and SSH
  - Specifically, we're going to implement ChaCha20
- ChaCha20 takes a 256-bit key as input
  - Represented as 8 32-bit **words** (a word is a fixed-size data unit, typically 32 bits)
- It also takes a second input: a 64-bit nonce
  - Represented as 2 32-bit words

State Initialization

- The initial state for ChaCha20 is a 4×4 matrix of 32-bit words constructed from:
  - σ = `"expand 32-byte k"` (128-bit constant, four 32-bit words)
  - k (256-bit key, eight 32-bit words)
  - j (64-bit counter, two 32-bit words)
  - n (64-bit nonce, two 32-bit words)
- Remember, this doesn't have to be implemented as a 2-dimensional array!

ChaCha20: State Matrix

4×4 matrix of 32-bit words (512 bits total)

σ_0	σ_1	σ_2	σ_3	constant `"expand 32-byte k"`
k_0	k_1	k_2	k_3	256-bit key
k_4	k_5	k_6	k_7	($8 \times$ 32-bit words)
j_0	j_1	n_0	n_1	64-bit counter (j_0, j_1); 64-bit nonce (n_0, n_1)

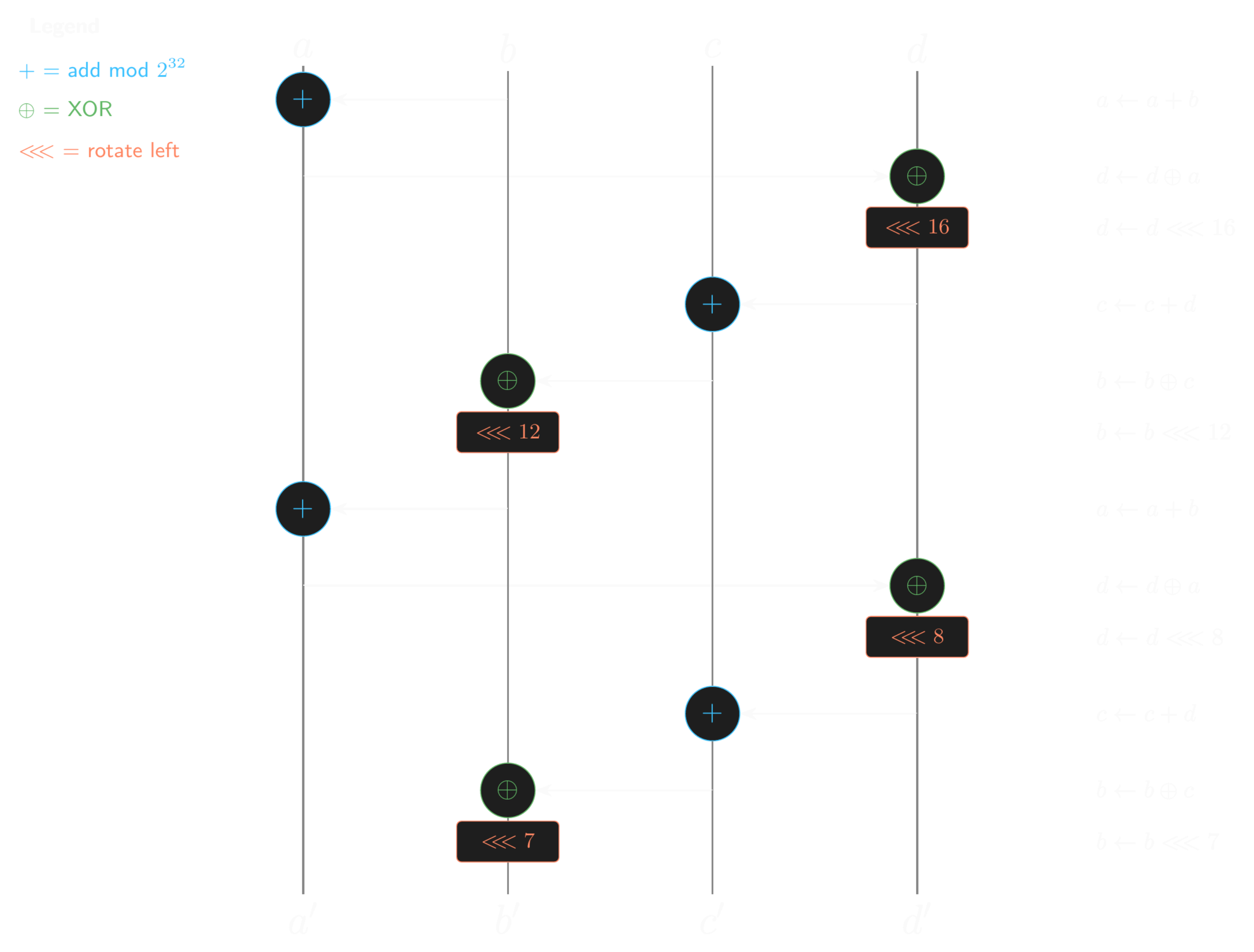
Permutation

- The permutation $\pi : \{0,1\}^{512} \to \{0,1\}^{512}$ is made up of a fixed number of **rounds**
  - ChaCha20 runs 20 rounds in total (hence the name)
- A simple quarter-round function is run four times per round
  - Odd rounds execute column-wise over the working state matrix
  - Even rounds execute diagonal-wise over the working state matrix
- The full spec can be found in RFC 7539
  - RFC 7539 – ChaCha20 and Poly1305
  - Note that the RFC uses a 32-bit counter and 96-bit nonce
- Finally, the initial state matrix is summed with the working state matrix
  - Without this step, the permutation could be inverted to recover the key!
  - The result is a 512-bit pseudo-random output block
- Setting the counter to j gets the $(j+1)$th output block
  - Random access and parallel computation are possible!

Quarter Round Function

- The quarter round function is made up of addition, XOR and rotation operations
- Easy to implement in software or hardware
- It's an ARX algorithm (Add-Rotate-XOR)
  - $x \leftarrow x \oplus (y \boxplus z) \lll n$
  - $\boxplus$ is modular addition, $\lll$ is rotate left
- Constant-time operations, no branching
  - Immune to timing attacks

ChaCha20: Quarter-Round Function



RC4

- Sometimes called ARCFOUR or ARC4 - “alleged” RC4
  - Leaked from, but never acknowledged by, RSA Security
  - The author (Ron Rivest) eventually confirmed it in a 2014 paper
  - Yes, that RSA Security! They’ll pop up a few more times in other lectures.
- Popular at the time!
  - Simple to implement, easy to apply
  - No need to worry about modes of operation, block sizes or padding
  - Very fast: comparable to AES, much faster than 3DES
- Some built-in OS CSPRNGs used RC4
- Let’s look at how it actually works

RC4: Key Scheduling Algorithm (KSA)

- RC4's internal state is a permutation of the integers $0, 1, \ldots, 255$
  - 256 bytes of state, often called the "S-box"
- The KSA initialises this permutation using the key k:
  - $\textbf{for } i = 0 \textbf{ to } 255 \textbf{ do: } S[i] \leftarrow i$
  - $j \leftarrow 0$
  - $\textbf{for } i = 0 \textbf{ to } 255 \textbf{ do:}$
    - $j \leftarrow (j + S[i] + k[i \bmod |k|]) \bmod 256$
    - $\mathrm{swap}(S[i], S[j])$

RC4: KSA Properties

- Only 256 swaps, regardless of key length
- Short keys (e.g. 40-bit WEP keys) repeat cyclically
  - Many elements barely move from their initial position

RC4: Keystream Generation (PRGA)

- Once the state is initialised, keystream bytes are generated one at a time:
  - $i \leftarrow 0,\ j \leftarrow 0$
  - $\textbf{for each output byte:}$
    - $i \leftarrow (i + 1) \bmod 256$
    - $j \leftarrow (j + S[i]) \bmod 256$
    - $\text{swap}(S[i], S[j])$
    - $\textbf{return } S[(S[i] + S[j]) \bmod 256]$
- Each output byte depends on the current state of the permutation
- The state evolves incrementally: each byte depends on all previous state
  - No random access, no parallelism (contrast with ChaCha20!)
- Beautifully simple: fits in a few lines of code

Why RC4 is Broken

- **Biased early outputs**: The KSA doesn't mix the state enough
  - The second output byte equals 0 with probability $\approx 1/128$ instead of $1/256$
  - This bias in the first bytes leaks information about the key
  - Even dropping the first 256 bytes doesn't fully fix the problem
- **No nonce input**: Same key always produces the same initial state and keystream
  - WEP concatenated the key with a 24-bit IV: only $2^{24} \approx 16$ million possible keystreams per key
  - This is exactly the many-time pad problem!
- **Statistical biases throughout**: Long-range correlations in the keystream
  - Practical plaintext recovery demonstrated against TLS (2013, 2015)
- Prohibited from use with TLS since 2015
  - Ironically, it was previously recommended as a workaround for the BEAST attack
- Don't use RC4!

“Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.”

John von Neumann

Randomness

A user's guide.

Classes of RNGs

- The secure PRGs we've discussed so far are also called CSPRNGs
  - Cryptographically Secure Pseudo-Random Number Generators
  - Be careful not to get these mixed up! Not all PRGs are secure
  - Picking the wrong kind of RNG for a task can have dire consequences
- True RNGs (TRNGs) are actually random, but expensive!
  - Usually used to seed a cheaper CSPRNG

Attacking RNGs

- A CSPRNG needs to do everything a normal PRNG can do
- That means passing rigorous statistical tests!
- E.g. the next-bit test: an attacker shouldn't be able to guess the next generated bit
  - ...even if they know every bit that came before it!
- In polynomial time, the best they can do is negligibly better than a 50-50 guess
  - Sound familiar?
- If you know the state of a PRNG, then you can figure out the next output
  - You might also be able to figure out past outputs (and the seed)
- A good CSPRNG must not reveal past outputs...
  - ...even if its state is fully or partially compromised!
- WPA2 vendors who used a hardcoded RNG seed got brute-forced like this...

What CSPRNG should we use?

- Most/all of you will probably use one in your projects!
- Probably some built-in secure random number generator (I hope...)
  - Python and Web Crypto both provide these
- Under the hood, most of these hook into the OS's RNG
  - E.g. `/dev/random` and `/dev/urandom` on Linux
  - These use real sources of entropy with a CSPRNG based on ChaCha20
- Please don't roll your own CSPRNG!
- Rolling your own PRNG for a game or other non-security context should be fine
  - But maybe not for research purposes or Monte Carlo simulations...
- If you want to try your hand at creating a PRNG, test it!
  - Use a well-known battery of statistical tests like TestU01
  - But remember, passing statistical tests doesn't imply security!
  - It's sine qua non: necessary but not sufficient

When RNG goes bad

Can you really trust your standard library?

Bad PRNGs

- Java (up until Java 17) uses an infamously poor kind of PRNG by default
- Linear congruential generators (LCGs) were already known to be poor…
- …but Java used one anyway (for the memes, presumably)
- LCGs are fast and don't need much memory
- Completely unsuitable for cryptography (of course)
- Problems get really obvious with multi-dimensional data

Linear Congruential Generators

- Let's build a PRNG!
- LCGs are old and well-known. They're also easy to implement!
- All you need to do is implement this recurrence relation:
  - $X_{n+1} = (a \cdot X_n + c) \mod m$ – set X_0 to the seed value
- The seed, multiplier and increment must be less than the modulus!
- There are a bunch of different subtypes
- Let's implement RANDU
  - Set $a = 65539, c = 0, m = 2^{31}$
- LCGs are sensitive to parameter choice
  - The LCG will repeat after a parameter-dependent period
  - The quality of the RNG will vary wildly depending on the parameters!
- RANDU used to be widely used, but it's laughably poor
  - May have wrecked lots of scientific papers

And this is why it's bad!

- If you plot RANDU output in 3D (taking consecutive triples as coordinates)...
  - The points fall on just 15 parallel planes!
  - This is because RANDU satisfies: $X_{n+2} = 6X_{n+1} - 9X_n \pmod{2^{31}}$
  - Every output is a linear combination of the previous two
- A truly random source would fill the cube uniformly
  - RANDU's output has massive, visible structure
- This is exactly the kind of pattern a statistical test (or an attacker) can detect
  - And why choosing good parameters for even a simple PRNG matters!

The Obligatory Java Bit

- It's easy to make fun of languages and standard libraries for using LCGs
  - Java usually gets laughed at for this
  - Java `Random` Documentation
- But it's important to note that LCGs are quick and easy to implement
  - And the right parameter choice can pass plenty of statistical tests
  - Good enough for plenty of use cases too
- The built-in RNG doesn't promise security!
  - So why would it be secure?
  - A good reminder to always read the docs before using RNG functions
- Pretty much every language will have CSPRNG implementations available
  - Use the right tool for the task at hand!

Better PRNGs

- Mersenne Twister is a decent general-purpose PRNG
- Huge period of $2^{19937} - 1$ (a Mersenne prime, hence the name)
- Relatively high memory requirements
- Passes most (but not all) standard test suites for statistical randomness
- Used as the default in many programming languages and libraries
- Even in some versions of Microsoft Excel
- Xorshift is a family of PRNGs
- Fast, very simple to implement, low memory requirements
- With some modifications, can pass most statistical tests
- But only once a non-linear step is added!
- Fits in a single screen's worth of C code

Quis custodiet ipsos custodes?

- We can’t talk about RNG in cryptography without mentioning that there have been several standards published and later withdrawn
  - Some that didn’t work as intended
  - And some that did work as intended, just not as expected…
- Most notoriously, Dual_EC_DRBG, published by NIST
  - DRBG? Deterministic Random Bit Generator, i.e. a PRNG
  - EC? Because it’s built using elliptic curves.
  - Usually associated with asymmetric cryptography.
  - Unusual for a PRNG…
- No security proof was published
  - Just a suggestion that it would be hard to crack
- You can probably guess what’s coming next!

The Dual_EC_DRBG Backdoor

- Turns out that the standard was mostly written by the NSA
- And the NSA secretly paid RSA Security $10m to include it as a default in their cryptographic library
- Why? Because Dual_EC_DRBG has a possible backdoor...
  - ...designed in such a way that only the NSA could confirm and exploit it
- Major embarrassment for NIST (and everyone else involved)
  - The standard has been withdrawn since
- OpenSSL's implementation, amusingly, never actually worked

Commit to the Bit

PRGs aren’t just for encryption.

Heads or Tails?

- Ever flipped a coin with someone?
- Easy for both parties to have mutual trust!
  - One person calls heads or tails
  - The other person flips the coin
- Both of them can hear the call and verify the result of the toss
  - No way for anyone to cheat it
  - Even with an unfair coin, you don't know if the call will be heads or tails
- This isn't so easy if you're not in the same place...
- If Alice tosses the coin knowing what Bob's guess is...
  - She can cheat!
- But if Bob only reveals his guess after he knows the result of the toss...
  - He can cheat!
- And neither of them can trust the result...

Bit Commitment

- Let's try to fix the coin toss protocol with some very practical™ cryptography!
- Bob needs to be able to commit to a guess - heads or tails, 0 or 1
  - Bob needs to be able to send a commitment string to Alice
  - It shouldn't give Alice any information about Bob's guess
    - This is the **hiding property**
- Even if Alice rigs the coin toss, she doesn't have a better chance of winning
  - Getting heads 100% of the time isn't useful
  - Bob has a 50% chance of guessing heads, so the odds are the same!
- Bob needs to be able to prove what his guess was after the toss
  - Bob sends Alice an opening string
  - Alice uses it to extract the guess from the commitment string
  - Bob shouldn't be able to change his guess!
    - This is the **binding property**

PRGs to the Rescue

- That's all well and good, but how can we actually implement it?
- Bob commits to a bit $b_0 \in \{0, 1\}$
- Alice picks a random $r \in \mathcal{R}$ and sends r to Bob
- Bob picks a random $s \in \mathcal{S}$ and computes $c \leftarrow \mathrm{com}(s, r, b_0)$
  - $\mathrm{com}(s, r, b_0) = G(s)$ if $b_0 = 0$
  - $\mathrm{com}(s, r, b_0) = G(s) \oplus r$ if $b_0 = 1$
- Bob sends the commitment string c to Alice
- After the toss, Bob sends his guess b_0 and the opening string s to Alice
- Alice verifies that $c = \mathrm{com}(s, r, b_0)$
  - If they match, she accepts that Bob's guess was b_0
  - Otherwise, she rejects it

Hiding Property

- The hiding property follows directly from PRG security!
  - $G(s)$ is computationally indistinguishable from a random $r \in \mathcal{R}$
  - And therefore $G(s) \oplus r$ is too
  - So Alice can't know what Bob's guess is
- That was easy!

Binding Property: Setup

- The binding property is a bit harder to show, and needs some constraints!
  - We require that $1/|\mathcal{S}|$ be negligible, i.e. that $|\mathcal{S}|$ be super-poly
  - And we require that $|\mathcal{R}| \geq |\mathcal{S}|^3$ (we'll see why in a moment)
- If Bob wants to cheat, he needs an opening string that can open to be 0 or 1
  - He needs to find $s_0, s_1 \in \mathcal{S}$ s.t. $c = G(s_0) = G(s_1) \oplus r$
  - $\Rightarrow G(s_0) \oplus G(s_1) = r$
- A "bad r" is one where some $s_0, s_1 \in \mathcal{S}$ exist to satisfy that equation.
- How many possible pairs of seeds can produce a bad r?

Binding Property: Counting Argument

- There are $|\mathcal{S}| \cdot |\mathcal{S}| = |\mathcal{S}|^2$ possible pairs of seeds
  - So there are, at worst, $|\mathcal{S}|^2$ bad r values
- How likely is it that Alice picks a bad r at random?
- $\frac{|\mathcal{S}|^2}{|\mathcal{R}|} < \frac{|\mathcal{S}|^2}{|\mathcal{S}|^3} = \frac{1}{|\mathcal{S}|}$
- $1/|\mathcal{S}|$ is negligible (by our constraint), so the binding property holds!
- The probability that Bob can cheat vanishes as the seed space grows

Bit Commitment

- This scheme isn't perfect...
  - You can attack it at the protocol level
  - But it's a pretty simple scheme and a neat use of PRGs
  - There are better bit commitment schemes out there if you ever need one!
- Lots of cryptographic primitives can be used in unexpected ways
  - In more complex schemes to achieve different goals
  - Or to build other cryptographic primitives
  - We'll see more of these in future lectures!

Conclusion

What did we learn?

So, what did we learn?

- Stream ciphers use a PRG to stretch a short key into a long keystream
  - $E(s, m) = G(s) \oplus m$
- The **security parameter** λ controls the level of security
  - Advantages must be **negligible** in λ; seed/key spaces must be **super-poly**
- A PRG is secure if its output is computationally indistinguishable from random
  - Formalised via the PRG security game (Experiment 0 vs Experiment 1)
- **Proof by reduction**: if the PRG is secure, the stream cipher is semantically secure
- PRGs can be composed in parallel or sequentially to produce longer outputs
  - The **hybrid argument** proves that security degrades linearly with n
- ChaCha20 is a modern, widely-used stream cipher (TLS, SSH)
  - Uses a nonce to safely encrypt multiple messages with the same key
- Not all PRNGs are created equal!
  - CSPRNGs for cryptography, regular PRNGs for everything else
  - Bad parameter choices and backdoors have caused real-world failures

For next time...

- Complete the challenges in this week's tutorial!
  - No grade this time, just bragging rights.
  - And stuff from the tutorials might be helpful later on...
- We'll take a look at block ciphers next week.
- Complete some assigned reading for next week:
- Chapter 7 of Crypto 101
- Sections 3.1 to 3.3 of Applied Cryptography
  - And 3.9 if you're interested in RC4
  - The rest of the chapter is interesting, but only if you've got time
  - I don't expect you to learn off proofs from the textbook!

Questions?

Ask now, catch me after class, or email eoin@eoin.ai